The order parameters characterization and their interaction theory of cuprate superconductors

Gu Jiapu

Chongqing 400039，In China

E-mail： gujiapu@cycdmy.edu.cn

Abstract

In this paper，we think that it is the key to find the high-T_c superconductivity mechanism to study all the order parameters and all their competitive relations fully in cuprate superconductors，and we analyse that there are six kinds of order parameters which should be paid attention to in superconducting state. The dynamic inequations of their competitive relations and the superconductivity theory are constructed in this paper. Directing a series of important experiment phenomenons of cuprate superconductors，we have a mathematical analysis systematically. It is shown that two factors act on the microscopic pairing mechanism simultaneously，the main factor is the magnetic resonance fluctuation energy between dynamic superconducting phase and antiferromagnetic background phase，the secondary factor is the phonon-electron interaction in cuprate superconductors.

Keywords：cuprate superconductors；order parameters characterization；interaction theory

1、 Introduction

There are two kinds of views about the high -T_c superconductivity mechanism in the present international physics research into cuprate superconductors，one view is generally believed that magnetic excitations play a fundamental role in the microscopic pairing mechanism because of the magnetic resonance mode [1，2] and the zinc or nickel element doping effect [3]，but the another viiew is believed that lattice vibrations play an important role in the microscopic pairing mechanism because of the isotope effect [4，5] in cuprate superconductors and the experiment observation on atomic scale [6].

But it's a pity，because no matter which kind of the above-mentioned two views，the physicists have not put forward an intact theory that can explain the superconductive phenomena in an all-round way yet at present. What is the reason?

In this paper，we think that it is the key to find the high -T_c superconductivity mechanism

to study all the order parameters and their competitive relations fully in the cuprate superconductors，because we find that the greatest disputes about pseudogap are the essences of order parameters and their competitive relations，usually different scientists have different views about them [7，8，9]， but there is no clear understanding so far，so it is the primary task to research the important order parameters and their interaction．

But what are the important order parameters of cuprate superconductors?we think that there are six kinds of order parameters which could not be neglected，their corresponding physical parameters are as follows：

(1) the condensation energy of antiferromagnetic background phase；

(2) the condensation energy of superconducting phase；

(3) the magnetic resonance fluctuation energy between dynamic superconducting phase and antiferromagnetic background phase；

(4) the long-range coulomb potential energy；

(5) the thermal vibration energy；

(6) the lattice vibration energy (phonon-electron interaction)．

The above-mentioned energies compete and restrict each other， when they meet a proper critical condition，the steady superconductive state then forms.

Below we will construct the mathematical expressions of all the order parameters in cuprate superconductors，and establish their competitive relation expressions further．

2、The basic assumption and concept

Because the superconducting electric current flows in one-dimensional vector direction symmetrically， in any section that is perpendicular to the current vector direction， it has the same energy function state． To describe the space translation of the energy function state，we should adopt the concept of energy density to express their corresponding order parameters．

We still realize that，the antiferromagnetic background phase and the superconducting phase are in ordered states of electric charge by magnetic interaction，so their order parameters have field vector properties and wave properties． In addition，their intensity of magnetic interaction relates to the electric charge surface density of ordered state， a certain mathematical relation should exist between them．

2.1 The concept of magnetic correlation intensity

In process of phase separation，the antiferromagnetic background phase and the superconducting phase are in ordered states of electric charge by magnetic correlation，and they behave as stripe phases usually，the stripe phases are thought to result from the reason that the densities of two kinds of electric charges are distributed and arranged in order with stripes form [10].

So we should have a concept of the magnetic correlation intensity to describe the correlation degree in the antiferromagnetic background phase and the superconducting phase.

Now we use the vector " $\vec{P}$ " to define the magnetic correlation intensity，Its corresponding scalar quantity is expressed with "*P*".

Because of the vector property，the intensity superposition of the magnetic correlation follows the vector compound theorem，the form of vector addition is as follows：

$$\vec{P} = \vec{P}_1 + \vec{P}_2 + \cdots + \vec{P}_n \qquad (1)$$

Now we can take advantage of the vector compound theorem of magnetic correlation intensity to express the coupling interaction between the antiferromagnetic background phase and the superconducting phase.

2.2 The mathematical relation between the magnetic correlation intensity and the electric charge surface density

We should set up a certain mathematical relation between the magnetic correlation intensity and the electric charge surface density.

As to cuprate superconductors，the electric charge surface density namely refers to the copper sites density in CuO_2 planes，among coppers ion，there exist strong magnetic correlation.

Now we suppose that there has a direct ratio relation between the magnetic correlation intensity value "*P*" of copper sites and the copper sites density value "*ρ*"，namely：

$$p = \alpha\rho \qquad (2)$$

The " *ρ*" namely represents the copper sites density.

And the "α" is a proportionality coefficient that relates to material medium.

The physical signification of eq.(2) is analogous to the theory " the field intensity in any

point on the charged surface has a direct ratio with electric charge surface density of oneself" on electrostatics.

2.3 The concept of the magnetic correlation energy density

On the basis of the concept of magnetic correlation intensity， we should still provide the concept of the magnetic correlation energy density，and set up the mathematical relation between them.

Being analogous to the concept of the magnetic field energy density in electromagnetism，we use the parameter "W" to define the magnetic correlation energy density， and we suppose that the mathematical relation between the magnetic correlation energy density and the magnetic correlation intensity has the form below by analogy：

$$W = \frac{1}{2}\beta P^2 \tag{3}$$

the "β" is a proportionality coefficient that relates to material medium.

Associating eq. (2) with (3)，the mathematical relation between the magnetic correlation energy density and the copper sites density has the form below：

$$W = \frac{1}{2}\alpha^2\beta\rho^2 \tag{4}$$

Appoint $k = \alpha^2\beta$, (5)

then，

$$W = \frac{1}{2}k\rho^2 \tag{6}$$

the "k" is a proportionality coefficient that relates to material medium.

2.4 The essence of the magnetic resonance mode

A magnetic resonance mode is known to exist in cuprate superconductors extensively [1，2]，in this paper we think that there exists a magnetic resonance correlation between the dynamical superconducting phase and the antiferromagnetic background phase，it is the essence of the magnetic resonance mode.

Now if we suppose P_1 is the antiferromagnetic background phase intensity， P_2 is the dynamical superconducting phase intensity，W_d is the coherent resonance energy density value of the two，that's：

$$W_d = \frac{1}{2}\beta(P_1 + P_2)^2 \tag{7}$$

Obviously on the numerical value，

$$W_d > \frac{1}{2}\beta P_1^2 + \frac{1}{2}\beta P_2^2 \tag{8}$$

Namely there has a energy redistribution by magnetic resonance mode.

3、 The mathematical expressions of the six kinds of energy factors in cuprate superconductors

Just as aforesaid，in high-temperature superconductors， there are six important energy factors as follows，now we calculate the mathematical expressions of their energy density：

(1) the antiferromagnetic background phase energy density (W_b)：

Because the forming of superconducting phase weakens the antiferromagnetic correlation， so in the regions of high charge density (stripes)， besides the mobile carriers(holes)， the background is the weakened antiferromagnetic phase.

Now we suppose the copper sites density in the antiferromagnetic background phase is ρ_s， and the corresponding correlation energy density is W_b， according to eq.(6)，it is ：

$$W_b = \frac{1}{2}k\rho_s^2 \tag{9a}$$

(2) the superconducting phase energy density (W_d)：

In the regions of high charge density (stripes)，suppose the mobile carriers(holes) gathering density is $\Delta\rho_s$， it is superposed on the background of the weakened

antiferromagnetic phase，and there are magnetic resonance coupling correlations between them，then according to eq. (7)， the superconducting phase energy density W_d is：

$$W_d = \frac{1}{2}k(\rho_s + \Delta\rho_s)^2 \qquad (9b)$$

(3) the magnetic resonance fluctuation energy density between dynamic superconducting phase and antiferromagnetic background phase (W_z)：

Because the carriers in superconducting phase are mobile，when they transport form a site to neighbor site， the new coupling and resonance forms．in the process，the magnetic resonance fluctuation energy between superconducting phase and antiferromagnetic background phase is transported at the same time．

Now suppose W_z is the magnetic resonance fluctuation energy density by transporting，then：

$$W_z = W_d - W_b = \frac{1}{2}k[(\rho_s + \Delta\rho_s)^2 - \rho_s^2] \qquad (9c)$$

(4) the long-range coulomb potential energy density (W_p)：

In process of phase separation，the condensation energy of superconducting state must overcome the long-range coulomb potential energy，suppose that before the phase separation，the long-range electrical potential energy distributes averagely in the material space．

Now we use “A” to represent the long-range coulomb potential energy，suppose the material space value is V，the long-range coulomb potential energy density value is W_p，then：

$$W_p = \frac{A}{V}. \qquad (9d)$$

Because doping impurities have little to influence the long-range coulomb potential energy “A” and the material space value “v”， we may regard the energy density value W_p as a constant that relates to the material．

(5) the thermal vibration energy density (W_t):

The condensation energy of superconducting state must still overcome the thermal vibration energy. In the macroscopical concept ，we use temperature concept “T” to measure the thermal vibration degree. Now we should calculate the corresponding energy density，and use “W_t“ to represent it.

According to the energy average theorem in thermodynamics，the thermal vibration energy that one mol solid has，namely each mol internal energy is 3RT，the “R” is the gas constant.

Suppose the investigation object namely the solid has N mol lattices，and its total volume is V，then the thermal vibration energy density is $3NRT/V$.

To a certain elected solid， the “N/V“ could be considered as a constant，suppose $\gamma = 3NR/V$ ，”γ“ is a constant naturally. Then the thermal vibration energy density W_t can write as：

$$W_t = \gamma T \quad , \tag{9e}$$

The modulus $\gamma = 3NR/V$ is a constant that relates to material.

(6) the lattice vibration energy density (W_s):

The lattice vibration mentioned here refers to the phonon-electron interaction in materials，its concept is the same as BCS superconductivity theory.

Now we suppose the parameter of the lattice vibration energy density in cuprate superconductors is W_s .

4、The superconducting mechanism and mathematical expressions

On the above-mentioned basis，we provide the superconducting mechanism as follows：

4.1 the mechanism of electron pairing：

We think that there are two kinds of energy factors playing important roles in the

mechanism of electron pairing：

one factor is the magnetic resonance fluctuation energy density between dynamic superconducting phase and antiferromagnetic background phase，its contribution to electron pairing is considered main in this paper.

Another factor is the lattice vibration energy，namely the phonon-electron interaction in materials，its contribution to electron pairing is considered secondary in this paper.

When the sum of the above-mentioned two kinds of energy density is greater than the sum of the long-range coulomb potential energy density and the thermal vibration energy density，then electron pairing forms．that is：

$$W_z + W_s > W_p + W_t$$

Namely：

$$\frac{1}{2}k[(\rho_s + \Delta\rho_s)^2 - \rho_s^2] + W_s > W_p + \gamma T \qquad (10)$$

4.2 the critical condition of superconducting stability：

The superconducting state remains steady on the premise that the antiferromagnetic background phase cannot be destroyed by other factors，because all the other energy factors are unfavorable to the antiferromagnetic background condensation..

So the critical condition of superconducting stability requires that the antiferromagnetic correlation energy must be greater than the sum of the magnetic resonance fluctuation energy，the long-range potential energy，the lattice vibration energy and the thermal vibration energy ．That is：

$$W_b > W_z + W_p + W_s + W_t$$

Namely：

$$\frac{1}{2}k\rho_s^2 > \frac{1}{2}k[(\rho_s + \Delta\rho_s)^2 - \rho_s^2] + W_p + W_s + \gamma T \qquad (11)$$

Combine eq.(10) with eq.(11) ，then：

$$\frac{1}{2}k\rho_s^2 > 2W_p + 2\gamma T \qquad (12)$$

The eq.(12) is the necessary condition of antiferromagnetic phase forming in the superconducting state.

In the investigation，we generally use the parameter of carrier concentration "x" to research，now we define the carrier concentration "x" as：

$$x = \frac{\Delta\rho_s}{\rho_s + \Delta\rho_s}$$

Suppose：

$$\rho_n = \rho_s + \Delta\rho_s,$$

The "ρ_n" corresponds to the copper sites amount in unit area of CuO_2 plane，among them the mobile carriers(holes) number is $\Delta\rho_s$，so the ratio of "$\Delta\rho_s/\rho_n$" is the carrier concentration.

Then：

$$\Delta\rho_s = x\ \rho_n$$
$$\rho_s = (1 - x)\ \rho_n$$

Replace them in eq.(10)(11)(12)，the Inequations have another forms：

$$T < -\frac{1}{2\gamma}k\rho_n^2(1-x)^2 + \frac{1}{\gamma}(\frac{1}{2}k\rho_n^2 - W_p + W_s) \qquad (13)$$

$$T < \frac{1}{\gamma}k\rho_n^2(1-x)^2 - \frac{1}{\gamma}(\frac{1}{2}k\rho_n^2 + W_p + W_s) \qquad (14)$$

$$\frac{1}{2}k\rho_n^2 + W_s > 3W_p + 3\gamma T \qquad (15)$$

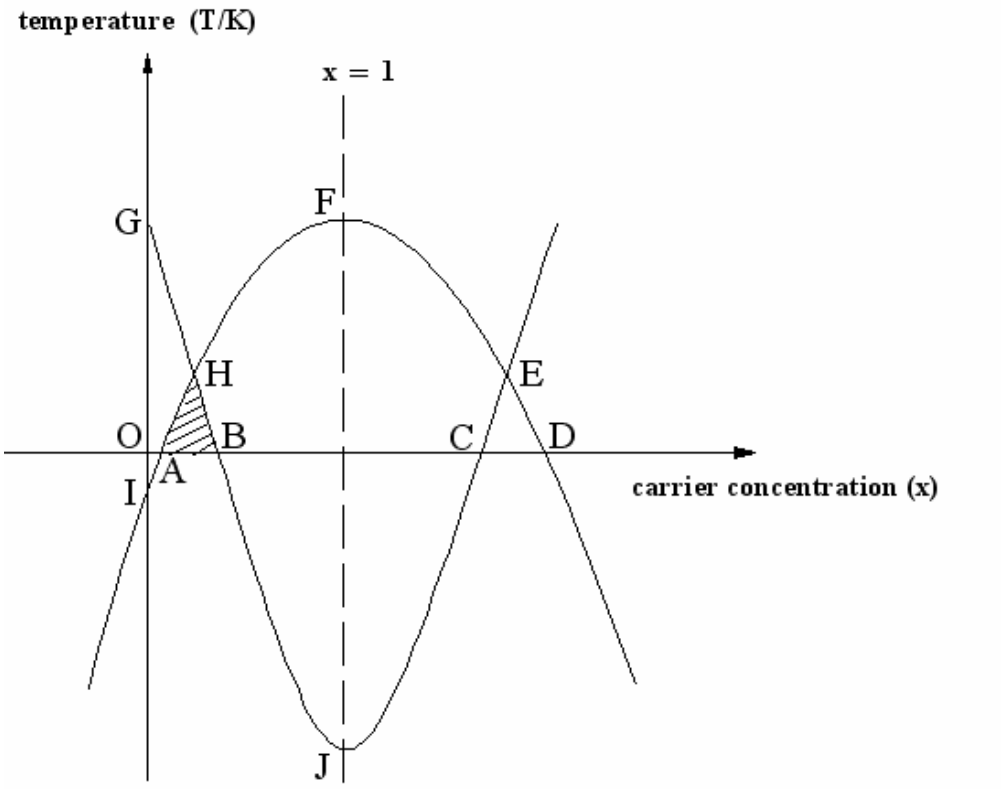
$(0 \le x \le 1,\ \ T \ge 0)$

Below we analyse the properties of cuprate superconductors based on the eq.(13)(14)(15).

5、The mathematical reasoning of weak doping and over-doping

characteristics

Now we set the carrier concentration "x" as abscissa，and set the temperature "T" as ordinate in a right-angled reference frame，then under the conditions "$0 \le x \le 1$，$T \ge 0$"，the area that Inequations (13)、(14) come into existence simultaneously is just the inclined shadow region "AHB" below ：



Fig．1 ： the change relation between temperature "T" and carrier concentration "x".

The inclined shadow region "AHB" is the realizable area of high-temperature superconductivity.

In Fig．1，the point coordinate of A、B、H is respectively：

$$A:(1-\sqrt{1-\frac{2(W_p-W_s)}{k\rho_n^2}}\ ,\ 0)$$

$$B:(1-\sqrt{0.5+\frac{W_p+W_s}{k\rho_n^2}}\ ,\ 0)$$

$$\mathrm{H}:(1-\sqrt{\frac{2}{3}+\frac{4\ W_s}{3\ k\rho_n^2}}\ ,\frac{k\rho_n^2}{6\gamma}-\frac{W_p}{\gamma}+\frac{W_s}{3\gamma})$$

From the Fig.1，we can learn these physical laws below that we think about：

(1) The variation tendencies of curve “AH”,” HB” reveal the weak doping and over-doping phenomena which exist in cuprate superconductors.

（2）The curve “GH” corresponds to the pseudogap in weak doping region. with the increasing of carrier concentration “x“ continuously， the pseudogap curve “GH” changes smoothly into the superconducting energy gap “HB” in over-doping area.

Because the pseudogap curve “GH” and the superconducting energy gap “HB” are two segments of the same smooth curve “GB”，so they have the same physical origin， namely the critical energy gap of the inequality (11) or (14)，it originates from the order parameters competitions between the antiferromagnetic correlation energy and the other parameters functions together by the magnetic resonance fluctuation energy，the long-range potential energy，the lattice vibration energy and the thermal vibration energy.

(3) The superconducting energy gap “AH” of the weak doping region becomes the non-superconductivity energy gap “HF” segment with the increasing of carrier concentration “x” continuously. They also have the same physical origin，namely the critical energy gap of the inequality (10) or (13). it originates from the order parameters competitions between two factors which are advantageous to superconductivity and other two factors which are unfavorable to superconductivity. The advantageous factors are the magnetic resonance fluctuation energy and the lattice vibration energy ，the unfavorable factors are the long-range potential energy and the thermal vibration energy.

As the “HF” segment comes from the smooth extension of the superconducting energy gap ‘AH”，so the instantaneous electron pairs are still be created in the non-superconducting area “BHF” that is below the segment “HF” and above the segment “HB”. But in this area the antiferromagnetic background phase is not steady，so the superconducting electron pair cannot maintain steadily. Only in the area that the antiferromagnetic background phase is steady relatively (below the segment “HB”)， the superconducting electron pairs and the superconducting current are steady. with the decreasing suddenly of the “HB” curve in the over-doping area， the short ordered stability of the antiferromagnetic background phase will

decrease suddenly and disappear step by step.

(4) From the Fig.1， we can know that the point “H” is the Intersection of the curve “GB” and the curve “AF”，it is the optimum doping point where the superconducting transition temperature has the highest peak value T_{cH} .

6、The mathematical reasoning of the antiferromagnetism enhancement effect

Now we study the mathematical relations between the critical temperature T_C and the total copper sites density ρ_n in unit area of CuO_2 plane when the carrier concentration x maintains invariable.

This kind of situation can be studied by Fig. 1 directly. In it ，if the value ρ_n increases to ρ_n'， the point A will move left to A'， the point B will move right to B'， and the point H will move vertically to H'，it is shown in the Fig. 2 below：

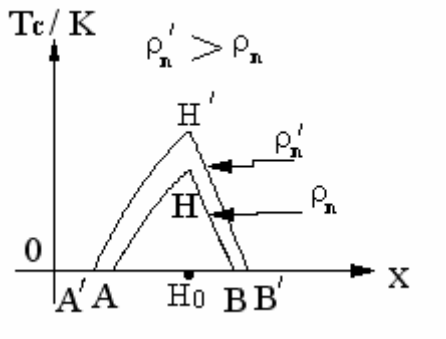


Fig. 2 ： the T_C -X function curve when the total density ρ_n changes

From the Fig. 2，we know that if the carrier concentration x maintains invariable，when the total density value ρ_n is increased by some ways such as ，the antiferromagnetic correlations are increased. At this time，the superconductivity critical temperature curve will move upward and expand the region of the carrier concentration that may achieve superconductivity early.

From the point coordinate expressions of A and B in Fig.1，we can get the extreme value of the point A、B，we have known that $k\rho_n^2 + 2W_s > 6W_p$ is the essential condition of superconducting state from the eq.(15)，if we suppose：

$$k\rho_n^2 >> W_p \geq W_s$$

then，

$$\frac{W_p \pm W_s}{k\rho_n^2} \to 0$$

So the point A、B have the extreme coordinate values below：

$$x_{A\min} = 0,$$
$$x_{B\max} = 1 - \sqrt{0.5} \approx 0.2929$$

Namely the carrier concentration has the following limit range in superconducting state：

$$x \in [0,\ 0.2929)$$

Now we study the other opposite condition，namely when the total density value ρ_n and the lattice vibration energy density W_s together become weaker and weaker，What will happen?

From the point coordinate expressions of A 、B and H in Fig.1，we know that：

If，

$$W_s \to 0,\ \ k\rho_n^2 \to 6W_p$$

Then the point A、B move and tend to the point H，the three points then get a convergence point H_0，that is：

$$H_0\text{:}\quad (1-\sqrt{\frac{2}{3}},\quad 0)$$

it approximates as follows，

$$H_0: \quad (0.1835, \quad 0)$$

7、The mathematical reasoning of the phonon-electron interaction and the isotope effect

We can rewrite the eq.(13)、(14) into the following forms：

$$T < -\frac{1}{2\gamma} k\rho_n^2 (1-x)^2 + \frac{1}{\gamma}(\frac{1}{2} k\rho_n^2 - W_p) + \frac{1}{\gamma} W_s \qquad (13\text{-}1)$$

$$T < \frac{1}{\gamma} k\rho_n^2 (1-x)^2 - \frac{1}{\gamma}(\frac{1}{2} k\rho_n^2 + W_p) - \frac{1}{\gamma} W_s \qquad (14\text{-}1)$$

$$(0 \le x \le 1, \quad T \ge 0)$$

If we neglect the phonon-electron interaction，namely only consider the magnetic resonance fluctuation energy between dynamic superconducting phase and antiferromagnetic background phase，then the eq.(13-1)、(14-1) change into the following forms：

$$T < -\frac{1}{2\gamma} k\rho_n^2 (1-x)^2 + \frac{1}{\gamma}(\frac{1}{2} k\rho_n^2 - W_p) \qquad (15)$$

$$T < \frac{1}{\gamma} k\rho_n^2 (1-x)^2 - \frac{1}{\gamma}(\frac{1}{2} k\rho_n^2 + W_p) \qquad (16)$$

$$(0 \le x \le 1, \quad T \ge 0)$$

Now we draw the superconducting critical region of eq.(15)、(16) by red dotted lines $A_z H_z B_z$，and draw the superconducting critical region of eq.(13-1)、(14-1) by black full lines AHB as the same as the Fig.1，they are compared in the Fig.3 below：

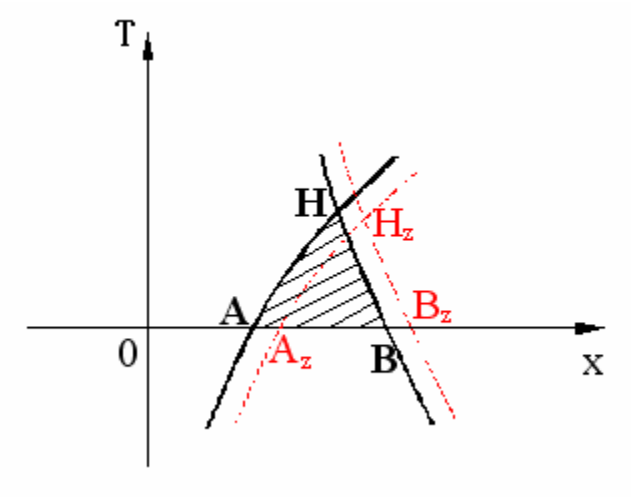


Fig. 3： the influence of the phonon-electron interaction on the superconducting critical temperature curves.

In the Fig.3，the point coordinate of A_z、B_z、H_z is respectively：

$$A_z:(1-\sqrt{1-\frac{2W_p}{k\rho_n^2}}\quad,\quad 0)$$

$$B_z:(1-\sqrt{0.5+\frac{W_p}{k\rho_n^2}}\quad,0)$$

$$H_z:(1-\sqrt{\frac{2}{3}}\quad,\frac{k\rho_n^2}{6\gamma}-\frac{W_P}{\gamma})$$

Now we can draw the following conclusions from the Fig.3：

(Ⅰ) Relative to the region $A_zB_zH_z$ in which we neglect the phonon-electron interaction，the real superconducting region ABH in which we consider the phonon-electron interaction shifts to the left. When the stronger the phonon-electron interaction is， the stronger the degree of the real superconducting region ABH shifting to the left is.

It is also shown that the enhancement of the phonon-electron interaction is advantageous to the enhancement of T_c value in weak doping area，but the T_c value in overdoping area is reduced relatively.

(Ⅱ) In the Fig. 3 we may know ， when the phonon-electron interaction appears to strengthen or weaken ， and when the energy density change value is ΔW_s ， the vertical

variation value of the T_c curve in weak doping area and over-doping area is $\frac{\Delta W_s}{\gamma}$; but this time the T_c change value of the optimum doping point H is only $\frac{\Delta W_s}{3\gamma}$ that thereupon rises or falls.

It is shown that the intensity change of the phonon-electron interaction (such as isotope effect) does much with the influence of the weak doping region and the over-doping region, but does little with the influence of the optimum doping point [11].

(Ⅲ)We can use the phonon-electron interaction energy in unit volume to express the energy density W_s of phonon-electron interaction. Now we suppose the vibration frequency of the phonon-electron interaction is υ, then the phonon-electron total energy of one crystal lattice is $h\upsilon$, the lattice volume is V_c, then the phonon-electron energy density W_s is:

$$W_s = \frac{h\upsilon}{V_c} \qquad (17)$$

From the eq.(13) (14), we know that the influence of phonon-electron interaction on the high temperature superconductivity is only depended on the phonon-electron energy density value W_s, but the value W_s is not only related to the lattice vibration frequency υ, but also related to the lattice volume V_c.

We know that external pressure may change the lattice volume V_c, so we think that the isotope effect is not only related to the lattice vibration frequency υ, but also related to the external pressure.

(Ⅳ)If we suppose the phonon-electron energy density value W_s becomes stronger and stronger by some reasons such as external pressure, the superconducting region ABH will shift to the left in Fig.3 continuously, then it will go through the following two kinds of states, such as the Fig.4 and Fig.5 show.

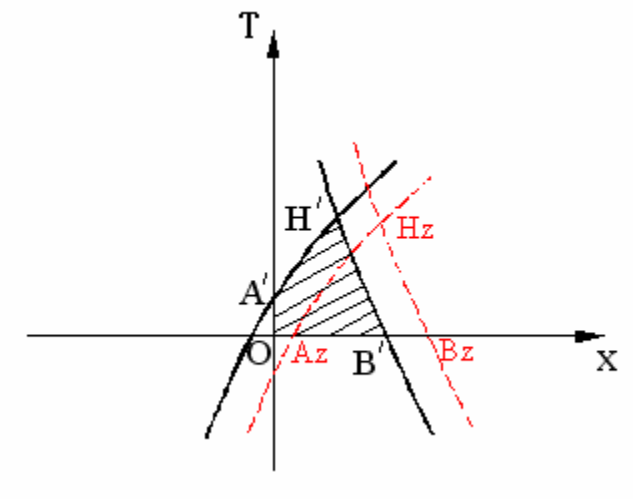


Fig.4 the superconducting state appears when the carrier concentration x=0，the inclined shadow region $OA'H'B'$ is the superconducting region.

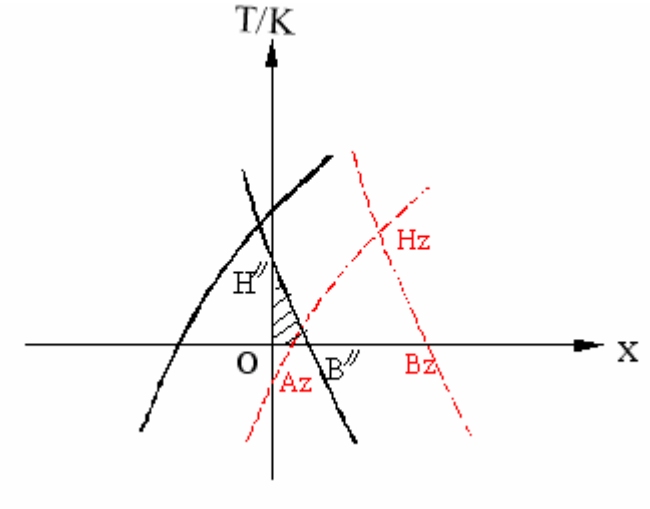


Fig.5 the superconducting critical curve becomes monotonic and falls，the inclined shadow region $OH''B''$ is the superconducting region.

In Fig.4， the phonon-electron energy density W_s has the following value ranges：

$$W_p < W_s < \frac{1}{6} k \rho_n^2$$

In Fig.5， the phonon-electron energy density W_s has the following value ranges：

$$\frac{1}{6}k\rho_n^2 < W_s < \frac{1}{2}k\rho_n^2 - W_p$$

In Fig.5， the point coordinate of H'' is as follows：

$$\mathrm{H}'': \quad (0, \frac{k\rho_n^2}{2\gamma} - \frac{W_p}{\gamma} - \frac{W_s}{\gamma})$$

So if the phonon-electron energy density W_s has the following value ranges：

$$W_s \geq \frac{1}{2}k\rho_n^2 - W_p$$

Then the phenomenon of superconductivity will vanish completely.

(Ⅴ) The phase diagram as the Fig.3 shows is the the superconducting state that we usually observe，and we know that the element substitution can enhance or weaken the ephonon-electron interaction and the antiferromagnetic correlation at the same time. now we study the influence of element substitution on the optimum doping point “H”.

For example， we study the superconducting transition temperature curves of Bi2Sr2CaCu2O8+δ with different amounts of cobalt substitution.

Because the cobalt substitution weakens the ephonon-electron interaction and the antiferromagnetic correlation at the same time. when the the cobalt substitution increases， the ephonon-electron interaction and the antiferromagnetic correlation are weakened to their minimum values， namely：

$$k\rho_n^2 \to 6W_p, \quad W_s \to 0$$

Then the optimum doping point “H” changes into the point H_0, namely：

$$H \to H_0 : (0.1835, \quad 0)$$

The point H_0 is the quantum critical point that we observe in In experiments [12,13].

It is shown in the Fig. 6 below：

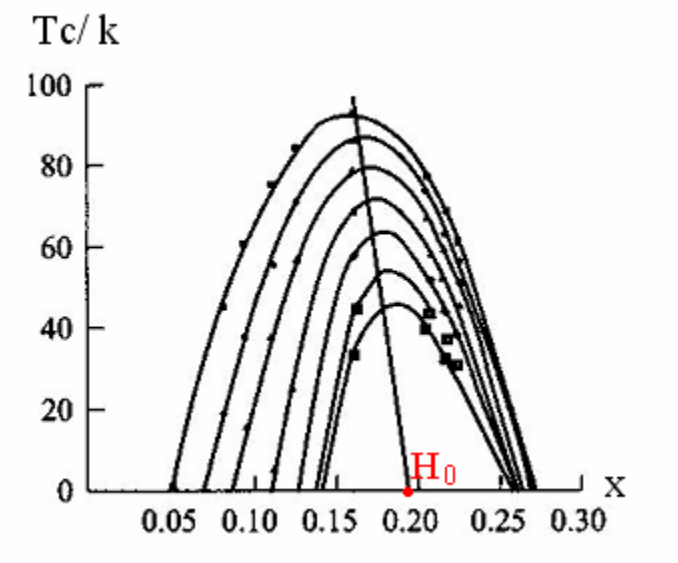


Fig.6 the superconducting transition temperature curves of Bi2Sr2CaCu2O8+δ with different amounts of cobalt substitution [13].

8、 Pressure effect and mathematical analysis

It is known that the pressure mainly compresses the crystal lattices volume to affect the superconductivity of cuprate superconductors，so under high pressure，we think that four parameters related to density concept will become greater， the four parameters are as follows：

(1) the total copper sites density in CuO_2 planes (ρ_n)；

(2) the lattice vibration energy density (W_s)；

(3) the carrier concentration (x)；

(4) the long-range coulomb potential energy density (W_p).

The above-mentioned parameters all become greater under high pressure，but how do they influence the superconductivity of cuprate superconductors?

Now combine the Fig.1 and the eq. (13) (14) ，we know that，

(Ⅰ) when the total copper sites density ρ_n increases， the superconductive

temperature critical curve （T_c） rises vertically．namely it is helpful to the superconductivity of cuprate superconductors．

（Ⅱ）when the lattice vibration energy density W_s increases，the superconductive region moves to the left in horizontal direction，and the critical peak value still rises continuously when $W_s \le \frac{1}{6} k\rho_n^2$，and begin to fall when $W_s > \frac{1}{6} k\rho_n^2$．

(Ⅲ) when the carrier concentration（x）increases，it is helpful to superconductivity in weak doping region，but it is not helpful to superconductivity in overdoping region．

（Ⅳ）when the long-range coulomb potential energy density W_p increases， the superconductive temperature critical curve （T_c） falles vertically．namely it is not helpful to the superconductivity of cuprate superconductors．

In sum，it seems very complicated when the above-mentioned four parameters act on the superconductivity of cuprate superconductors at the same time ．but we can make use of the peak value T_{cH} to judge the comprehensive result that four parameters act on at the same time．

Because，

$$T_{cH} = \frac{k\rho_n^2}{6\gamma} - \frac{W_p}{\gamma} + \frac{W_s}{3\gamma}$$

So we use the $\frac{dT_{cH}}{dP}$ to judge the influence of high pressure on superconductivity of cuprate superconductors．

Because，

$$\frac{dT_{cH}}{dP} = \frac{d\left(\frac{k\rho_n^2}{6\gamma} + \frac{W_s}{3\gamma}\right)}{dP} - \frac{d\left(\frac{W_p}{\gamma}\right)}{dP}$$

The $\frac{dT_{cH}}{dP}$ is a positive number or a negative number， it relates to the crystal

structure of cuprate superconductors.

We know that cuprate superconductors are usually udivided into p-type superconductors and n-ype superconductors，p-type superconductors have the crystal structure of Pyramids or octahedrons shape， n-ype superconductors have the crystal structure of square planes shape.

And we know that the copper ion in the center of square planes structure suffers the greater crystal field potential well than the copper ion in the center of octahedrons or in the bottom center of Pyramids structure，so the copper sites in p-type superconductors are easier to condense and move than the carriers in n-ype superconductors under pressure.

so we think as follows，

(1) To p-type superconductors，

$$\frac{d\left(\frac{k\rho_n^2}{6\gamma}+\frac{W_s}{3\gamma}\right)}{dP}>\frac{d\left(\frac{W_p}{\gamma}\right)}{dP},$$

namely $\frac{dT_{cH}}{dP}>0$，so high pressure can usually raise the T_c value. but after being up to a maximum，it will begin to reduce to zero because of the moving to left continuously of the superconductive region.

(2)To n-type superconductors，

$$\frac{d\left(\frac{k\rho_n^2}{6\gamma}+\frac{W_s}{3\gamma}\right)}{dP}\le\frac{d\left(\frac{W_p}{\gamma}\right)}{dP},$$

namely $\frac{dT_{cH}}{dP}\le 0$，so high pressure can not usually raise the T_c value， on the contrary， the T_c value nearly keeps intact and finally the high pressure reduces the T_c value.

9、Conclusion

In above-mentioned contents， a series of important phenomena of cuprate superconductors are explained，we may summarize the following conclusions and viewpoints from this paper：

（Ⅰ）it is the key to find the high Tc superconductivity mechanism to study all the order parameters and all their competitive relations fully in cuprate superconductors. The competitive relations of the order parameters cause the complicated dynamics structure.

（Ⅱ）it is shown that two factors act on the microscopic pairing mechanism simultaneously，one main factor is the magnetic resonance fluctuation energy between dynamic superconducting phase and antiferromagnetic background phase，the secondary factor is the phonon-electron interaction in cuprate superconductors.

(Ⅲ) the total copper sites density (ρ_n)、the lattice vibration energy density (W_s)、the carrier concentration (x) of superconductive CuO_2 planes are three important factors of affecting the superconductive critical temperature T_c.

(Ⅳ) under high pressure，the long-range coulomb potential energy density (W_p) increases and has different influences on p-type superconductors and n-ype superconductors because of the difference of their crystal field potential well.

（Ⅴ）the carrier concentration (x) in superconductive region has the theoretical extreme range： $x \in [0,\ 0.2929)$， and has the quantum critical point nearby “0.19”， its precise value is $H_0: (0.1835,0)$.

(Ⅵ) in usual conditions，choose the suitable cuprate materials that have great phonon-electron interaction， on the basis， enhance the total copper sites density (ρ_n) as possible as it can by no pressure way，and dope the optimum carrier concentration (x)， this is the effective way to raise the superconductive critical temperature T_c.

Finally we believe that the theoretical research and the practical application of cuprate superconductors will obtain considerable progress further.

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About author：

Gu Jiapu，male，a free scholar，ever graduated from North University of China in 1999，now lives in chongqing in china. The main research direction：high temperature superconductivity，dark matter and dark energy and the general theory of relativity and quantum mechanics.